

Network of domain walls on soliton stars

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We explore the idea of a network of domain walls appearing on the surface of a soliton star. We show that for a suitable fine-tuning among the parameters of the model we can find localized fermion zero modes only on the network of domain walls. In this scenario, the soliton star becomes unstable and decays into free particles before the cold-matter upper mass limit is achieved. However, if fermions do not bind to the network of domain walls, the network becomes neutral, imposing a new lower bound on the charge of the soliton star, slightly raising its critical mass.

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I. INTRODUCTION

According to general relativity, if a star has a sufficiently small mass, it can reach final equilibrium as a white dwarf or neutron star. On the other hand, if the mass of the collapsing portion of a star is greater than the *cold matter* upper mass limit M_c , equilibrium can never be achieved and complete gravitational collapse will occur. Usually, the final stage of such a collapse is the formation of a black hole. For normal matter, the mass limit M_c is approximately equal to five solar masses M_\odot (at zero angular momentum) [1,2].

In the context of soliton stars, it is possible to have objects with a larger mass without gravitational collapse. The subject of soliton stars has been introduced in Ref. [3] (see also [4,5]). They are new types of cold stable stellar configurations, and depending on the theory, one may have $M_c \sim 10^{15} M_\odot$. Recently [6] soliton star models were used in order to explain current observational data, which favor the possible existence of a single supermassive object at the center of our galaxy, other than a supermassive black hole.

In this paper we investigate how the structure of a soliton star can be affected by the entrapment of another object on its surface. The idea that we explore is similar to the case of lower-dimensional domain walls living inside domain walls [7–13], which is inspired by the mechanism used to build superconducting cosmic strings [14]. The basic mechanism consists in the formation of a scalar condensate due to the spontaneous symmetry breaking of a scalar field, invariant under $U(1)$ gauge symmetry, confined to the cosmic string. In the context of brane world scenarios renewed recently in [15,16] (see also [17,18] for earlier ideas related to this topic), one assumes that the fields describing the fundamental particles, except the graviton (see Ref. [19] for trapping of the graviton; see also Ref. [20] and references therein for investigations in supergravity), are confined to a 3-brane (a four-dimensional manifold describing our universe) embedded in a higher-dimensional spacetime. The scalar fields that live in the 3-brane can also generate a scalar condensate by spontaneous symmetry breaking of some discrete symmetry. When a discrete symmetry is spontaneously broken, for instance a Z_2 symmetry, a *domain wall* can form inside the

3-brane [21]. For other Z_N symmetries, with $N > 2$ we have the possibility of having intersection of domain walls forming junctions, and then a network of domain walls.

In order to investigate the entrapment of a network of domain walls by a soliton star, we consider the possibility of a spherical [22,23] two-dimensional wall to entrap wall segments that form a network. This possibility may give rise to a network of domain walls [24,25] to live at the surface of a standard soliton star [3]. We examine this idea starting with an appropriate model, described by three real scalar fields, introduced according to the lines of Ref. [25]. The model comprises several parameters, and below we show that depending on the type of fine-tuning used to adjust these parameters, we can produce either heavier soliton stars or instability that will ultimately induce their complete decay.

Our work is organized as follows. In Sec. II, we present the model and we investigate the entrapment of the network of domain walls. In Sec. III, we study in detail the presence of localized zero modes on the network and its consequences. Comments and conclusions are given in Sec. IV, which closes our work. Our notation is standard, and we use dimensional units such that $\hbar = c = 1$, and metric tensor with signature $(+ - - -)$.

II. THE MODEL

In Ref. [25], we have investigated the possibility of a domain wall to entrap a network of domain walls. This investigation was inspired in Ref. [24], which dealt with the idea of building a planar network of domain walls. The model investigated therein engenders the Z_3 symmetry, which is the simplest symmetry that enables the presence of junctions of domain walls. The presence of triple junctions in supersymmetric models engendering the Z_3 symmetry was investigated in Refs. [26–29], with several distinct motivations. For instance, the basic idea of Ref. [26] was to present the Bogomol'nyi equation for the triple junction, showing that the planar junction of domain walls only preserves 1/4 supersymmetry of the model, which is in contrast with the case of a single domain wall, which is known to preserve 1/2 supersymmetry of the corresponding model.

The presence of planar triple junctions may allow the tiling of the plane with a regular hexagonal network, and this issue was further examined in Ref. [28], and also in [24,29]. See Refs. [30–34] for several other issues related to this subject. As one knows, the most efficient way to tile the plane with regular polygons is obtained by the regular hexagonal network, and this makes the Z_3 symmetry preferable, among many other possibilities. Very interestingly, the Z_3 symmetry also appears as the center of the $SU(3)$ group, which governs the symmetry of QCD, the field theory that describes strong interactions; see, e.g., Ref. [35] and references therein for recent investigations on this subject. Interest in walls and in wall junctions widens when one recalls that the low-energy world volume dynamics of branes in string/ M theory may be described by standard models in field theory [36–40]. Furthermore, this interest goes beyond the context of high-energy physics: For instance, it also appears in ferroelectric materials where walls and wall junctions spring as stable structures in many different ferroelectric crystals [41].

The underlying symmetry of the standard model of elementary particles contains the $SU(2) \times SU(3)$ group, and this is the basic inspiration to consider a model that engenders the $Z_2 \times Z_3$ symmetry, that is, the discrete counterpart of $SU(2) \times SU(3)$. For this reason, we follow the lines of Ref. [25] to introduce the model

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - V(\sigma, \phi, \chi) \\ & + i \bar{\psi} \gamma^\mu \partial_\mu \psi + m \bar{\psi} \psi - f \sigma \bar{\psi} \psi + \lambda (\phi + \chi) \bar{\psi} \psi. \end{aligned} \quad (1)$$

This model contains three real scalar fields that couple among themselves via the potential $V(\sigma, \phi, \chi)$, which is introduced below. Also, there is a massive Dirac fermion ψ that couples to the scalar fields via the Yukawa couplings $f \sigma \bar{\psi} \psi$ and $\lambda (\phi + \chi) \bar{\psi} \psi$; see Ref. [42] for information on the behavior of fermions in the background of topological defects generated by real scalar fields.

The potential is chosen to provide the standard spherical soliton star [3] with a network of domain walls on its surface. The sigma field has to give rise to the host domain wall, which should entrap the other two fields, which have to engender the Z_3 symmetry. The model should be able to describe a spherical soliton star via the scalar field σ , by breaking its Z_2 symmetry under the shift $\sigma \rightarrow \sigma - (1/2)\sigma_0$, to entrap the other two fields (ϕ, χ) with a Z_3 symmetry on its surface. We get to this model by considering the potential

$$\begin{aligned} V = & \frac{1}{2} \mu^2 \sigma^2 (\sigma - \sigma_0)^2 + \lambda^2 (\phi^2 + \chi^2)^2 - \lambda^2 \phi (\phi^2 - 3\chi^2) \\ & + \left[\lambda \mu \left(\sigma - \frac{1}{2} \sigma_0 \right)^2 - \frac{9}{4} \lambda^2 \right] (\phi^2 + \chi^2). \end{aligned} \quad (2)$$

Here $\sigma=0$ and $\sigma=\sigma_0$ are the true and false vacua corresponding to the standard soliton star.

The scenario for a soliton star to entrap a network of domain walls should constrain the symmetries of the poten-

tial. According to the Euler theorem, to tile the sphere with regular polygons of the same type we need (a) 4, (b) 8, or (c) 20 triangles, (d) 6 squares, or (e) 12 pentagons. These tiling show the cases where the polygon edges end in three-junctions (a,d,e), four-junctions (b), and five-junctions (c). These configurations are topologically equivalent to the five Platonic solids, namely the tetrahedron $\{3,3\}$, octahedron $\{3,4\}$, icosahedron $\{3,5\}$, cube $\{4,3\}$, and dodecahedron $\{5,3\}$; see Refs. [43,44] (here the notation $\{M,N\}$ stands for regular M -gons and N junctions). In this sense, we can use a potential with Z_N symmetry between the fields ϕ and χ (that must inhabit the interior of the domain wall formed by the other field σ) to describe locally N junctions on the surface of a sphere [25] (the soliton star surface). We have three possibilities of choosing N ($N=3,4,5$), and we choose $N=3$, that is, the Z_3 symmetry. While this is the minimal possibility for the appearance of junctions, it is also the center of the $SU(3)$ group, that is, the group of the strong interactions. Another possibility with the symmetry breaking $Z_3 \rightarrow Z_2$ was considered in [25] in order to describe a tiling with 12 pentagons and 20 hexagons, which resembles the fullerene structure of 60 carbon atoms. The main mechanism for a domain wall to entrap a network of domain walls was explored in Ref. [25]. The key point here is that, as we shall see, on the surface of the soliton star $\sigma \simeq (1/2)\sigma_0$, and at this place, the remaining fields (ϕ, χ) develop nonzero vacuum expectation value (VEV) (condensate) with three different phases that contribute to form domain-wall three-junctions and then a network. We summarize this phenomenon as follows.

We use the equations of motion to see that in the false and true vacua $\sigma=\sigma_0$ and $\sigma=0$, respectively, the fields (ϕ, χ) turn out to be zero. For the scalar fields $\phi, \chi=0$ and the fermion field $\psi=0$, the theory (1) and (2) allows the field σ to form a soliton solution. We note that such a solution can be found by using the following first-order differential equation:

$$\frac{d\sigma}{dR} = \mu \sigma (\sigma - \sigma_0) = W_\sigma, \quad (3)$$

where $W = \mu(\sigma^3/3 - \sigma^2\sigma_0/2)$ can be seen as a superpotential that defines the potential $V(\sigma, 0, 0) = (1/2)W_\sigma^2$ [29]. Equation (3) can be integrated to give the solution

$$\sigma = \frac{\sigma_0}{2} \left[1 - \tanh \frac{\mu \sigma_0 (R - R_0)}{2} \right]. \quad (4)$$

This solution shows that at the surface ($R \simeq R_0$), the σ field goes to $(1/2)\sigma_0$. It represents approximately [45] a spherical wall (the soliton star surface) with surface tension [29]

$$t_h \simeq |W(\sigma_0) - W(0)| = \frac{1}{6} \mu \sigma_0^3. \quad (5)$$

In the regime of $\sigma \simeq (1/2)\sigma_0$, the remaining scalar fields (ϕ, χ) engender Z_3 symmetry, and describe three-junctions of domain walls which allow the formation of a network

[24]. In the thin wall approximation, each segment of the network can be represented by a domain wall (kink) solution of the explicit form

$$\begin{aligned}\phi &= -\frac{3}{4}, \\ \chi &= \frac{3}{4}\sqrt{3}\tanh\sqrt{\frac{27}{8}\lambda}(z-z_0).\end{aligned}\quad (6)$$

The other segments are obtained by rotating the (ϕ, χ) plane by 120° and 240° , respectively. Below, we shall investigate how the domain-wall segments in the network may have normalizable fermionic zero modes. The issue of whether or not we have normalizable zero modes on the junction (in [46], the non-normalizable zero mode on Bogomol'nyi-Prasad-Sommerfield (BPS) junctions, was found in the supersymmetric context) does not affect our discussion since the junction here is itself an approximately zero-dimensional object, and then with a negligible Fermi gas if fermions may bind to it.

Neutral network on fermion soliton stars

The spherical wall described by the σ field is a nontopological soliton. Thus it requires a conserved Noether charge of bosonic and/or fermionic origin to stabilize it. According to the model used in this paper, we are choosing fermionic charges to stabilize the soliton star. The soliton star is then a fermionic soliton star.

In this case, in the false vacuum σ_0 the mass of the fermion field goes effectively to zero if we assume

$$m - f\sigma_0 = 0. \quad (7)$$

The soliton star becomes stable due to a three-dimensional Fermi gas pressure (see, e.g., Refs. [47,48] for a similar discussion in the context of Fermi balls). This is the scenario of the standard nontopological structure called the fermion soliton star [4] (similar studies for scalar soliton stars were considered in [5]), and now we have to consider the effects of the other two fields ϕ and χ that we have already introduced in the model.

Regardless of the type of network that inhabits the soliton star, in general the total energy of the system is $E = E_n + E_s + E_k$, which will be given below. Since we are assuming that the fermions are in the interior of a spherical false vacuum of radius R in (3,1) space-time dimensions, then the kinetic energy of the confined fermions is

$$E_k = \frac{Q}{R}, \quad Q = \frac{1}{2}\left(\frac{3}{2}\right)^{5/2} \pi^{1/3} N^{4/3}, \quad (8)$$

where N is the fermion number. The surface of the soliton star contains the surface energy

$$E_s = \alpha R^2, \quad \alpha = 4\pi t_h, \quad (9)$$

where t_h is the surface tension of the soliton star. Finally, the energy of the nested network is

$$E_n = \beta R, \quad \beta = n\xi t_n, \quad (10)$$

where n and $\xi R = d$ are the number and length of the segments in the network, $t_n = (27/8)\sqrt{3/2}\lambda$ is the tension of each segment [24,25], and ξ is a real constant. The total energy is therefore

$$E = \alpha R^2 + \beta R + \frac{Q}{R}. \quad (11)$$

We minimize this energy (11) by using $\partial E/\partial R = 0$, which allows us to obtain the critical radius

$$R_0 = \frac{1}{6\alpha} \left[A^{1/3} + \frac{\beta^2}{A^{1/3}} - \beta \right], \quad (12)$$

where

$$A = 54Q\alpha^2 - \beta^3 + 6\sqrt{3}\sqrt{Q(27Q\alpha^2 - \beta^3)}\alpha. \quad (13)$$

We notice that in this scenario Q turns out to have a lower bound, in order for a critical radius to exist. It is given by

$$Q \geq \left(\frac{\beta}{3}\right)^3 \frac{1}{\alpha^2}, \quad (14)$$

which is nonvanishing for $\beta \neq 0$. In the limit of large N , the total energy of the configuration is

$$E \sim \alpha R_0^2 = \alpha \left(\frac{Q}{2\alpha}\right)^{2/3} \sim N^{8/9}. \quad (15)$$

We note that the exponent of N is less than unity. This means that for large N , the energy of the soliton is always less than the energy of the free particles, thus the stability of the soliton star is ensured. This is the same limit obtained in [3,4]. We conclude that a neutral network does not contribute in the large- N limit.

Let us now consider the network contribution to the cold-matter upper mass limit M_c in this scenario. We may estimate such a limit by simply equating the radius (12) to the Schwarzschild radius $R_s = 2GM$. First of all, we note that the minimum of the energy (11) at the critical radius (12) is the soliton mass $M = 3\alpha R_0^2 + 2\beta R_0$, where we have written the charge Q in terms of R_0 , that is, $Q = (2R_0\alpha + \beta)R_0^2$. Now we set $R_0 \sim R_s$, which leads to

$$\begin{aligned}M_c &\sim \frac{1}{12\alpha} \left(\frac{1+4\beta G}{G^2} \right) \\ &\sim (48\pi t_h G^2)^{-1} + [12\pi(t_h/t_n)G]^{-1}(n\xi),\end{aligned} \quad (16)$$

where n, ξ are numbers inherent to the type of network one is considering. They contribute to higher *excitations* due to the network at the surface of the soliton star. Notice that for $n = 0$ (no network), M_c reduces to the first term ("the fundamental state"), which is the same result found in Ref. [3]. Since for $n \neq 0$ the last term is positive, we conclude that the network raises the standard value of M_c and then yields

heavier soliton stars. For a typical energy scale of the order of GeV and $\lambda = \mu = 1$, we find the values for the tensions $t_h = (1/6)(30 \text{ GeV})^3$ and $t_n = 6\sqrt{3}/2(3/4)^2(\text{GeV})^2$ (here we have chosen $\sigma_0 = 30 \text{ GeV}$). Now using the fact that $\sqrt{G} = l_p \approx 10^{-33} \text{ cm}$ (the Planck length) and $M_\odot \sim 10^{33} \text{ g}$, we find

$$M_c \sim (30 \text{ GeV})^{-3} l_p^{-4} + (30 \text{ GeV})^{-1} l_p^{-2} (n\xi) \sim (10^{15} + 10^{-23}) M_\odot \quad (17)$$

Here we have dropped the factor $n\xi$ in the last step, since it can change the mass of the soliton star by at most two orders of magnitude. This is because $\max(n) = 30$ (i.e., the dodecahedron $\{5,3\}$ case) and $\max(\xi) = 2\pi$ (i.e., the largest arc on the sphere). This result allows us to conclude that in this model the network raises slightly the mass of the standard soliton star. The radius size of this object for $\sigma_0 = 30 \text{ GeV}$ is $R_0 \sim 10^2$ light years.

III. FERMION ZERO MODES ON THE NETWORK

In this section, we discuss how to adjust the parameters in our model in a way such that the fermions prefer to migrate from the false vacuum to the network. It is perfectly possible that the effective fermion mass inside the network, which, in turn, is at the surface of the soliton star, goes to zero if we assume the fine-tuning

$$m - f\sigma_s - \lambda v_s = 0, \quad (18)$$

where $\sigma_s = (1/2)\sigma_0$ is the value of the σ field at the surface of the soliton star, and $v_s = 3/4$ is the norm of the vector field (ϕ, χ) inside the network.

The way we couple fermions to the fields (ϕ, χ) in the model (1) and (2) is standard, although it does not preserve the Z_3 symmetry that governs the (ϕ, χ) portion of the model, which allows the formation of a regular network of domain walls as shown in Ref. [24]. With the alternative coupling $\lambda\chi(\chi^2 - 3\phi^2)\bar{\psi}\psi$, the Z_3 symmetry should be preserved, but in this case non-normalizable zero modes such as $\psi(z) = \exp(\pm C \text{sech}^2 z)\epsilon_\pm$, with $m = (1/2)f\sigma_0$ (C is a real constant) would be present. Similar conclusions were found in [46]. There, it was found that an object preserving the Z_3 symmetry (BPS junctions, in global supersymmetry) gets non-normalizable zero modes as well.

The effective fermion mass given in terms of the background solution (6) confined to the surface of the soliton star can be written as

$$M_F(z) = m - \frac{1}{2}f\sigma_0 - \left(\frac{3}{4}\right)\lambda \left[1 - \sqrt{3}\tanh\sqrt{\frac{27}{8}}\lambda(z-z_0)\right]. \quad (19)$$

In the above equation, we see that $M_F(z_0)$ recovers the left-hand side of Eq. (18), allowing us to conclude that the effective fermion mass $M_F(z)$ goes to zero inside the network. This means that the fermions prefer to live inside the network ($z \approx z_0$) rather than in the false vacuum $\sigma = \sigma_0$. The fermionic zero mode inside the network is described by

$$\begin{aligned} \psi(z) &= e^{ix^\nu p_\nu} \exp\left(\pm \int_0^z M_F(x) dx\right) \epsilon_\pm \\ &= e^{ix^\nu p_\nu} e^{\pm(m-f\sigma_s-\lambda v_s)z} \\ &\quad \times \left[\cosh\sqrt{\frac{27}{8}}\lambda(z-z_0)\right]^{\pm 1/\sqrt{2}} \epsilon_\pm. \end{aligned} \quad (20)$$

Here we have used $\gamma^\nu p_\nu \psi = 0$ and $\gamma^z \epsilon = \pm i \epsilon_\pm$ to solve the Dirac equation for the zero mode. ($\nu = 0, 1$ is the tangent frame. Also, z is the coordinate transverse to each domain wall at the surface of the soliton star and ϵ_\pm is a constant 2-spinor.) From Eq. (18), we see that the second exponential factor in Eq. (20) does not contribute to the zero mode. Finally, we find that the only normalizable zero mode is

$$\psi(z) = e^{ix^\nu p_\nu} \left[\cosh\sqrt{\frac{27}{8}}\lambda(z-z_0)\right]^{-1/\sqrt{2}} \epsilon_\pm. \quad (21)$$

This shows that there are localized chiral fermion zero modes [12,42,48,49] in each domain-wall segment of the network. In other words, there are localized massless fermions only on the network.

Charged network on neutral soliton stars

Now we are ready to present another scenario. Let us suppose that all the fermions feel an attractive strong force so that they are forced somehow to migrate from the false vacuum to the nested network on the surface of the soliton star. This is exactly what is experienced by the fermion zero modes that we have just treated. In this case, we should replace the former tridimensional Fermi gas by another Fermi gas, approximately one-dimensional, which spreads along the network conserving the fermion number N . Thus, all we have to do in the investigation done in Sec. II A is to replace Eq. (11) by

$$E = \alpha R^2 + \beta R + \frac{\gamma}{R}, \quad \gamma = \frac{\pi N^2}{4\xi}, \quad (22)$$

where we have used the expression for one-dimensional Fermi gas [12],

$$E = \frac{\pi N^2}{4L}, \quad (23)$$

where $L = d = \xi R$ is the length of the segments in the network. In the large- N limit, we can also use Eq. (15) to obtain

$$E \sim \alpha R_0^2 = \alpha \left(\frac{\gamma}{2\alpha}\right)^{2/3} \sim N^{4/3}, \quad (24)$$

where we have set $Q \rightarrow \gamma$. Now, since the exponent of N is greater than unity, the energy of the soliton for large N is always larger than the energy of the free particles, thus the stability of the soliton star is no longer ensured.

IV. CONCLUSIONS

In this paper, we have presented a model that can describe two totally different scenarios, described in Sec. II A and in Sec. III A. In the first scenario, although the neutral network slightly increases the cold-matter upper mass limit of the standard soliton star, it imposes a new lower bound on the charge of the fermionic soliton star. In the second scenario, the entrapment of the Fermi gas inside the network changes significantly the behavior of the nontopological soliton, destabilizing the soliton star. In the equilibrium stage of the fermion migration, the false vacuum becomes neutral and the network becomes charged with massless fermions. This result led us to the conclusion that in the scenario of Sec. III A, the formation of a charged network at the surface of the soliton star can destabilize it. This is because the localized fermion zero modes on the network make it energetically favorable for the soliton star to decay into free particles. They can decay fast before the cold-matter upper mass limit M_c is achieved. These two scenarios can be thought of as two possible different experiences that a soliton star can suffer in the cosmological evolution. Suppose that the soliton star and the network appear at distinct critical temperatures T_s and T_n , respectively (see Ref. [10] for a study about critical temperature and defects formation). Thus for T_s sufficiently larger than T_n , the soliton star can start its formation process by collapsing, approaching the Schwarzschild radius before the strong force due to the fermionic zero mode of the network acts. If the network appears later, it will be “frozen” together with the other constituents of the soliton star due to a strong gravitational force, leading to the scenario of Sec. II A. On the other hand, if $T_s \sim T_n$, one may induce the scenario described in Sec. III A. In this scenario,

if we consider that the soliton star may form cosmic objects, we see that the more the soliton star has the experience of Sec. III A, the less these cosmic objects are formed in the universe.

The new objects that spring in the present work may be of particular interest to astrophysical and cosmological applications, as for instance in the recent investigations introduced in Refs. [6,25,44]. Furthermore, these objects provide alternative routes to the Fermi balls examined in Refs. [47,48], and as such they may furnish distinct view of the scenario discussed therein. Another line of investigation may follow Ref. [50], in which the behavior of surface current-carrying domain walls was examined, the current being of bosonic origin, which appears in a model engendering the $U(1) \times Z_2 \rightarrow U(1)$ symmetry, with $U(1)$ global. The model of Ref. [50] is inspired by the case of cosmic strings introduced in Ref. [14]. Our model provides another possibility, where fermionic current may flow on the surface of a wall that hosts a network of domain walls, so that we could ask how the presence of the nested network would contribute to change the fermionic behavior in the wall. Some of these issues are presently under consideration, and we hope to report on them in the near future.

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